Eur. Phys. J. A **14**, 77–80 (2002)
DOI 10.1140/epja/iepja1314

THE EUROPEAN PHYSICAL JOURNAL A

Self-affine multiplicity fluctuation of target residues in relativistic nuclear collisions at a few GeV to a few hundred GeV

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Received: 22 October 2001
Communicated by R.A. Ricci

Abstract. Self-affine multiplicity scaling is investigated in the framework of two-dimensional factorial moment methodology using the concept of the Hurst exponent (H). Our investigation on experimental data of the target-evaporated slow particles emitted in ^{32}S-AgBr interactions at 200 AGeV and ^{28}Si-AgBr interactions at 14.5 AGeV reveals that a better power law behavior is exhibited in self-affine analysis than self-similar analysis. This work shows a clear evidence of self-affine target fragmentation.

PACS. 25.75.-q Relativistic heavy-ion collisions – 25.70.Mn Projectile and target fragmentation – 24.60.Ky Fluctuation phenomena

1 Introduction

Particle physics has a long history of intense experimental and theoretical activity in search of scale invariance and fractality in multihadron production processes, for short also called "intermittency". These investigations cover all types of reactions ranging from e^+e^- annihilation to nucleus-nucleus collisions, up to the highest attainable energies. The concept of intermittency was first introduced by Bialas and Peschanski [1] in the field of high-energy physics through the analysis of the distribution of produced particles from a cosmic-ray event [2] of heavy-ion interaction. Intermittency is signaled by the power law behavior of the scaled factorial moments with increasing spatial resolution of the particle detection procedure. The unique feature of this moment is that it can detect and characterize the non-statistical density fluctuations in particle spectra, which are intimately connected with the dynamics of particle production. Subsequent analysis of various collision processes, in the available accelerator energy range, also reveals the same basic observations of Bialas and Peschanski [3] and thereby intermittency appears to be a general phenomenon in multiparticle production in relativistic high-energy interactions.

Most of the investigations have been carried out in the pionisation region with the common belief that pions are the most informative about the collisional dynamics. However, the physics of relativistic interactions is not yet conclusive and therefore we need all probes to analyze thoroughly and meaningfully the experimental data.

Here we deal with target fragments from nuclear interactions. The nuclear fragmentation or "evaporation" data are very conveniently provided by nuclear emulsion experiments. The characteristics about the dynamics of target-evaporated low-energy particles are not revealed as yet in detail. The black tracks are identified as target evaporation particle in a model referred to as the "evaporation model". In the evaporation model, the particles corresponding to "shower" and "grey" tracks are emitted from the nucleus very soon after the instant of impact, leaving the hot residual nucleus in an excited state. Emission of particles from this state takes place relatively slowly. In order to escape from this residual nucleus, a particle must await a favourable statistical fluctuation, as a result of random collisions between the nucleons within the nucleus, which leads the particle close to the nuclear boundary, traveling in an outward direction. After the evaporation of this particle a second particle is brought to the favourable condition for evaporation and so on, until the excitation energy of the residual nucleus is so small that the transition to the ground state is likely to be affected by the emission of rays. The evaporation model is based on the assumption that statistical equilibrium has been established in the decaying system and the directions of emission of the evaporating particles are distributed isotropically. However, the concept of the evaporation model has not been universally accepted. Barkas suggested [4] that mechanisms other than the evaporation process must also be considered to explain the emission of heavy fragments from excited nuclei, as the evaporation process cannot explain the emission of all the heavy fragments. Different experimental data also indicate the existence of the non-

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equilibrium nature of processes to be responsible for the emission of slow, target-associated particles.

Most of the analysis has been performed in one-dimensional space, though it is not at all sufficient for extracting the proper fluctuation patterns of the real three-dimensional process [4,5]. The analysis should be done in higher dimension to reduce the error due to dimensional reduction. Even in two-dimensional intermittency analysis [6] reported so far the usual procedure was to divide the corresponding phase space subsequently into subcells by shrinking equally in each dimension and all works in both dimensions conclude that the pionisation process is self-similar. However, the phase space in high-energy multiparticle production is anisotropic as indicated by Van Hove [7]. Most of the present analyses were inadequate to some extent in a sense that they overlooked the anisotropy of phase space and consequently the possibility of having a self-affine fluctuation pattern consistent with the nature of the phase space structure.

Previously we worked on target-associated particles in high-energy interaction in two dimensions considering the θ-ϕ phase space as an isotropic one [8]. From our intutive feeling that the multiplicity fluctuation of target residues may also produce a self-affine behavior we have analyzed in this work the behavior of target-evaporated slow particles produced in^{32}S-AgBr interactions at 200 AGeV and ^{28}Si-AgBr ones at 14.5 AGeV considering the anisotropy of phase space and this study has given positive evidence of the self-affine behavior rather than self-similar in case of emission of target-associated particles.

2 Experimental details

Stacks of G5 nuclear emulsion plates have been horizontally exposed to a ^{32}S beam at 200 AGeV from CERN SPS at CERN and a ^{28}Si beam of energy 14.5 AGeV from the Alternating Gradient Synchrotron at Brookhaven National Laboratory (BNL AGS) [9]. A Leitz Metaloplan microscope with a 10× objective and 10× ocular lens provided with a semi-automatic scanning stage was used to scan the plates. Each plate was scanned by two independent observers to increase the scanning efficiency. The final measurements were done using an oil immersion 100× objective. The measuring system fitted with it has 1 μm resolution along the X and Y axes and 0.5 μm resolution along the Z-axis.

After scanning, the events were chosen according to the following criteria:

i) The incident-beam track should not exceed 3° with respect to the main beam direction in the pellicle. It is done to ensure that we take the real projectile beam.
ii) Events showing interactions within 20 μm from the top and bottom surface of the pellicle were rejected. It is done to reduce the loss of tracks as well as to reduce the error in the angle measurement.
iii) The tracks of the incident particle, which induce interactions, were followed in the backward direction to ensure that the beam is a projectile beam starting from the beginning of the pellicle.

According to the emulsion terminology [10] the particles emitted after interactions are classified as

a. Black particles: they are target fragments with ionization greater than or equal to $10I_0$, I_0 being the minimum ionization of a singly charged particle. Their range is less than 3 mm, their velocity less than $0.3c$ and their energy less than 30 MeV. c is the velocity of light in vacuum.
b. Grey particles: they are mainly fast target recoil protons with energy up to 400 MeV. They have ionization $1.4I_0 \leqslant I < 10I_0$. Their range is greater than 3 mm and their velocity is $0.7c \geqslant V \geqslant 0.3c$.
c. Shower particles: the relativistic shower tracks with ionization I less than or equal to $1.4I_0$ are mainly produced by pions and are not generally confined within the emulsion pellicle.
d. The projectile fragments are a different class of tracks with constant ionization, long range and small emission angle.

To ensure that the targets in the emulsion are silver or bromine nuclei, we have chosen only the events with at least eight heavy ionizing tracks of (black + grey) particles.

According to the above selection procedure we have chosen 140 events of ^{32}S-AgBr interactions at 200 AGeV and 352 events of ^{28}Si-AgBr interactions at 14.5 AGeV. The emission angle (θ) was measured for each track by taking the coordinates (X_0, Y_0, Z_0) of the interaction point, coordinates (X_1, Y_1, Z_1) at the end of the linear portion of each secondary track and coordinate (X_i, Y_i, Z_i) of a point on the incident beam.

It is worthwhile to mention that the emulsion technique possesses very high spatial resolution, which makes nuclear emulsion a very effective detector for studying the self-affine behavior of multiplicity fluctuation.

A procedure has been adopted for self-affine analysis of factorial moments, where the size of the elementary phase space cells can vary continuously. Considering the two-dimensional case and denoting the two phase space variables as x_1 and x_2, the factorial moment of order q may be defined as [1]

$$F_q(\delta x_1,\delta x_2) = \frac{1}{M'}\sum_{m=1}^{M} \frac{n_m(n_m-1)............(n_m-q+1)}{\langle n_m\rangle^q}\,, \tag{1}$$

where $\delta x_1\ \delta x_2$ is the size of a two-dimensional cell, n_m is the multiplicity in the m-th cell and M' is the number of two-dimensional cells into which the considered phase space has been divided.

To fix δx_1, δx_2 and M', we consider a two-dimensional region $\varDelta x_1\ \varDelta x_2$ and divide it into subcells with widths

$$\delta x_1 = \varDelta x_1/M_1\,, \tag{2}$$

$$\delta x_2 = \varDelta x_2/M_2\,, \tag{3}$$

in the x_1 and x_2 directions where $M_1 \neq M_2$ and $M' = M_1 \cdot M_2$.

Here M_1 and M_2 are the scale factors that satisfy the equation

$$M_1 = M_2^H\,, \tag{4}$$

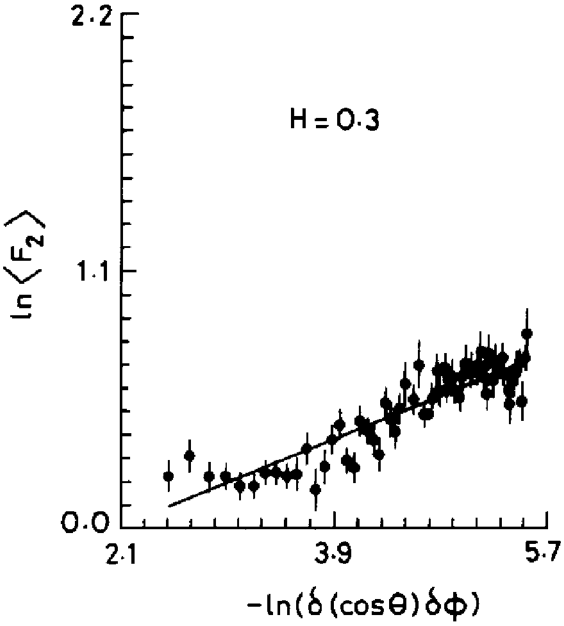

Fig. 1. Plot of $\ln\langle F_2\rangle$ *vs.* $\ln(\delta(\cos\theta)\delta\varphi)$ for $q = 2$ and $H = 0.3$ in the two-dimensional space of $\cos\theta$ and φ for ^{32}S-AgBr interactions at 200 AGeV.

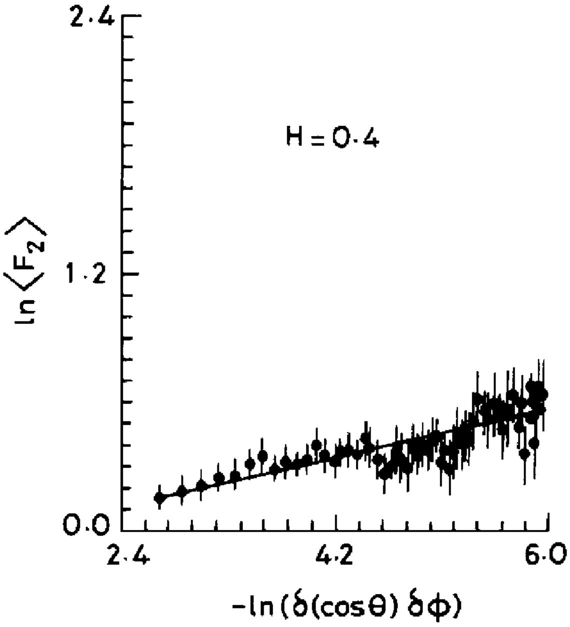

Fig. 2. Plot of $\ln\langle F_2\rangle$ *vs.* $\ln(\delta(\cos\theta)\delta\varphi)$ for $q = 2$ and $H = 0.4$ in the two-dimensional space of $\cos\theta$ and φ for ^{28}Si-AgBr interactions at 14.5 AGeV.

where $0 < H \leqslant 1$ is called the Hurst exponent [11]. It is a parameter characterizing the self-affinity property of dynamical fluctuations. It is clear from eq. (4) that the scale factors M_1 and M_2 cannot simultaneously be integer, so that the size of the elementary phase space cell should be able to take continuously varying values.

The following method [11] has been adopted for performing analysis with non-integral values of the scale factor (M). For simplicity consider the one-dimensional space (y) and let

$$M = N + a\,, \tag{5}$$

where N is an integer and $0 \leqslant a < 1$. When we use the elementary bin of width $\delta y = \Delta y/M$ as "scale" to "measure" the region Δy, we get a number N of bins and a smaller bin of width $a\Delta y/M$ left. Putting the smaller bin at the last (or first) place of the region and doing the average with only the first (or last) N bins, we have

$$F_q(\delta x) = \frac{1}{N}\sum_{m=1}^{N} \frac{\langle n_m(n_m-1)..........(n_m-q+1)\rangle}{\langle n_m\rangle^q}\,. \tag{6}$$

M determined by (5) can be any positive real number and so can vary continuously.

If the target fragmentation process is self-affine, then the factorial moment should obey a power law behavior of the form $F_q(\delta x) \propto (\delta x)^{-\alpha_q}$ as $\delta x \to 0$. The power $\alpha_q > 0$ is a constant at any positive integer q and is called intermittency exponent.

We have performed our study in the two-dimensional cosine of the emission angle-azimuthal angle (w.r.t. the beam direction) space. The cosine of the emission angle region used is -1 to $+1$ and that of the azimuthal region is 0 to 2π. As the shape of this distribution influences the scaling behavior of the factorial moments, we have used the "cumulative" variable [4] $X_{\cos\theta}$ and X_ϕ instead of $\cos\theta$ and ϕ. The corresponding region of investigation for both the variables then becomes (0,1). The cumulative variable $X(x)$ is given by the relation

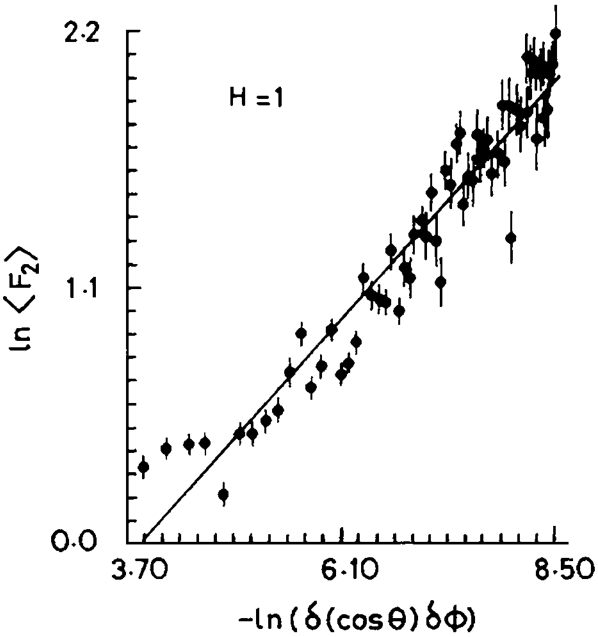

Fig. 3. Plot of $\ln\langle F_2\rangle$ *vs.* $\ln(\delta(\cos\theta)\delta\varphi)$ for $q = 2$ and $H = 1$ in the two-dimensional space of $\cos\theta$ and φ for ^{32}S-AgBr interactions at 200 AGeV.

$$X(x) = \int_{x_1}^{x} \rho(x')\partial x' \Big/ \int_{x_1}^{x_2} \rho(x')\partial x'\,, \tag{7}$$

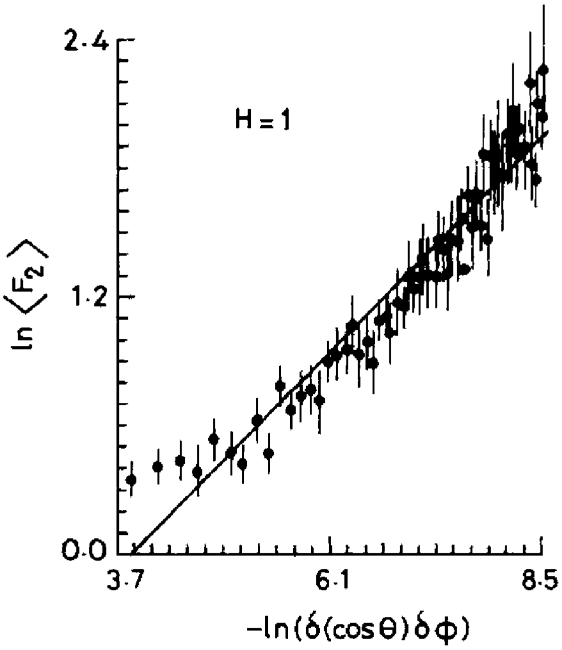

Fig. 4. Plot of $\ln\langle F_2\rangle$ *vs.* $\ln(\delta(\cos\theta)\delta\varphi)$ for $q = 2$ and $H = 1$ in the two-dimensional space of $\cos\theta$ and φ for ^{28}Si-AgBr interactions at 14.5 AGeV.

Table 1. Values of χ^2/DOF and intermittency exponents for two particular values of the Hurst exponent H in ^{32}S-AgBr interactions and ^{28}Si-AgBr interactions.

Primary beam	Value of H	χ^2/DOF	Intermittency exponent
Sulphur	0.3	0.965	0.21 ± 0.014
	1	1.27	0.43 ± 0.016
Silicon	0.4	0.418	0.13 ± 0.011
	1	0.994	0.42 ± 0.015

where x_1 and x_2 are two extreme points in the distribution $\rho(x)$, between which X varies from 0 to 1.

To probe the exposed anisotropic structure of the phase space, we have calculated the factorial moments for the 2nd order with the varying values of the Hurst exponents. The partition numbers in longitudinal and transverse directions are chosen as $M_P = 7, 8, 9, ...70$ and $M_n = M_P^H$.

We have not considered the first few data points $M_P = 2, 3, 4, 5, 6$ in order to reduce the effect of momentum conservation [12] which tends to spread the particle in opposite directions and thus reduce the value of the factorial moments. This effect becomes weaker as M increases.

3 Result and discussion

We have plotted the natural logarithm of ($\delta X_{\cos\theta}$, δX_ϕ) along the X-axis and the natural logarithm of the average value of the factorial moments of 2nd order ($\ln\langle F_2\rangle$) along the Y-axis for different Hurst exponents starting from $H = 0.1$ to $H = 1$ by steps of 0.1. In order to find the partition condition at which the anisotropic behavior is best revealed, we have performed the linear best fit. For which χ^2 per degrees of freedom is calculated.

For sulphur the best linear fit occurs at $H = 0.3$ (fig. 1) and in the case of silicon the plot corresponding to $H = 0.4$ (fig. 2) gives the best linear fit. The best linear fit proves that the anisotropic behavior is best revealed at $H = 0.3$ for the sulphur beam and at $H = 0.4$ for the silicon beam. Both for sulphur and silicon it is seen that when $H = 1$, χ^2/DOF is high as is also evident from the convex shape of the curve (fig. 3 and fig. 4). Table 1 represents the values of H, χ^2/DOF and the intermittency exponent for the sulphur and silicon beams. It is interesting to observe that the values of the intermittency exponents decrease with decreasing energy. Thus, the above analysis provides the first evidence of the self-affine behavior of the target fragmentation in relativistic and ultra-relativistic nucleus-nucleus collisions and this information is extremely useful for understanding the emission phenomena of target fragments.

The authors are grateful to Prof. P.L. Jain, Buffalo State University, USA for providing us with the exposed and developed emulsion plates used for this analysis. We also gratefully acknowledge the financial help from the University Grants Commission (India) under the COSIST programme.

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